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Comment on Kaluza-Klein spectrum of gauge fields in the bigravity model

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Abstract: We study behavior of bulk gauge field in the bigravity model in which two positive tension AdS_4 branes in AdS_5 bulk are included. We solve the equations of motions for Kaluza-Klein modes and determine the mass spectrum. It is shown that unlike the case of graviton, we find no ultralight Kaluza-Klein modes in the spectrum.

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Over the past few years, theories with extra spatial dimensions have received much attention because they potentially solve long standing problems such as the hierarchy problem, as was originally suggested by Antoniadis, Arkani-Hamed, Dimopoulos and Dvali [1, 2]. Afterwards Randall and Sundrum found a new solution to the hierarchy problem [3]. In addition, they showed that gravity could be localized on a brane with infinite extra dimension [4]. Since then, various types of models including gravity have been studied.

Recently Kogan et al. proposed a new "bigravity" model [5]. The model has only two positive tension AdS_4 branes in AdS_5 bulk and no negative tension branes. Owing to the absence of negative tension branes, the model does not have the ghost fields which appeared in the previous "bigravity" scenario [6] and also in the quasi-localized gravity model [7]. But interestingly in the model of [5] the bounce of the warp factor mimics the effect of a negative tension brane and thus gives rise to an anomalously light graviton Kaluza-Klein mode. Moreover it is possible in this model to circumvent the van Dam-Veltman-Zakharov no go theorem [8, 9] about the non-decoupling of the extra polarization states of the massive graviton [10]. Similar discussions have recently appeared in [11]–[14].

In the previous work [15], we studied behavior of the zero mode of the bulk gauge field in this bigravity model in more detail. In this paper, we try to understand the massive (Kaluza-Klein) modes in the model. Scalar and fermion fields in this model have already been discussed extensively [16, 17]. It has already wellknown that bulk gauge fields in the ordinary Randall-Sundrum model have quite a specific behavior. The reason consists in the conformal property of the gauge field action, i.e., the 4-dimensional kinetic term of the gauge field is not warped, which leads us to the fact that the zero mode of the bulk gauge field in the RS model is not localized on a brane. It is flat in the extra dimension. Many people have tried to resolve this issue [18]–[24], but no one has not succeeded completely yet. In that meaning, it is important to investigate various aspects of bulk gauge fields in different higher dimensional models including the bigravity model.

Following the work by Kogan et al. [5] we shall consider a five dimensional anti-de Sitter spacetime (AdS_5) with a warp factor

$$ds^2 = \Omega^2(w)(\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu + dw^2)\,, \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$. The metric $\bar{g}_{\mu\nu}(x)$ denotes an AdS_4 background. The warp factor $\Omega(w)$ in the conformal coordinate w is given by

$$\Omega(w) = \frac{1}{\cosh(kz_0)}\frac{1}{\cos\tilde{k}(|w|-\theta)}\,, \tag{2}$$

where $\tilde{k} \equiv k/\cosh(kz_0)$ and $\tan(\tilde{k}\theta/2) = \tanh(kz_0/2)$, while $\tanh(kz_0) \equiv kV_1/|\Lambda|$. k is the curvature of AdS_5 defined through $k \equiv \sqrt{-\Lambda/24M^3}$. Λ is the five dimensional cosmological constant which is negative. V_1 and V_2 are tensions of the 3-branes at the orbifold fixed points, $w=0$ and $w=w_L$, respectively.

We are interested in behavior of the bulk gauge field in this background metric. Let us start with the following five dimensional action;

$$S_{GF} = -\frac{1}{4}\int d^4x \int_{-w_L}^{w_L} dw \sqrt{-G} G^{MN} G^{RS} F_{MR} F_{NS}\,, \tag{3}$$

where $M, N, R, S = 0, 1, 2, 3, w$ and $F_{MN} = \partial_M A_N - \partial_N A_M$. $A_M(x^\mu, w)$ is the bulk U(1) gauge field. The extension to non-abelian gauge field is straightforward. G_{MN} is the five dimensional metric defined through eq. (1). The equations of motions are given by

$$\partial_M(\sqrt{-G} G^{MN} G^{RS} F_{NR}) = 0\,. \tag{4}$$

We solve the equations with the gauge condition $A_w = 0$ and as usual, we expand the field $A_\mu(x^\mu, w)$ into zero mode and Kaluza-Klein modes:

$$A_\mu(x, w) = \sum_{n=0}^{\infty} a_\mu^{(n)}(x)\rho^{(n)}(w)\,. \tag{5}$$

Then the equation of motion for the bulk gauge field reduces to [15]

$$\partial_w(\Omega(w)\partial_w \rho^{(n)}) + m_n^2 \Omega(w)\rho^{(n)} = 0\,. \tag{6}$$

Here m_n denote the mass eigenvalues.

The zero mode of the gauge field ($m_n = 0$) is of the form

$$\rho^{(0)}(w) = \frac{a}{\tilde{k}}\cosh(kz_0)\sin\tilde{k}(w-\theta) + b\,, \tag{7}$$

where a and b are integration constants. We discussed some properties of the zero mode in the previous paper [15] so that we don't mention it any more.

On the other hand, the KK modes satisfy the following equations;

$$\begin{aligned} &\frac{d^2\rho^{(n)}}{dw^2} + \tilde{k}\tan(\tilde{k}(w-\theta))\frac{d\rho^{(n)}}{dw} + m_n^2\rho^{(n)} = 0 \qquad \text{for} \quad w > 0\,, \\ &\frac{d^2\rho^{(n)}}{dw^2} + \tilde{k}\tan(\tilde{k}(w+\theta))\frac{d\rho^{(n)}}{dw} + m_n^2\rho^{(n)} = 0 \qquad \text{for} \quad w < 0\,. \end{aligned} \tag{8}$$

The general solution of eqs. (8) is given in terms of hypergeometric functions [15];

$$\begin{aligned} \rho^{(n)}(w) = {}& c_1 F\Big(\alpha_n, \beta_n, \frac{1}{2}; \sin^2(\tilde{k}(|w|-\theta))\Big) + \\ & + c_2|\sin\tilde{k}(|w|-\theta)|F\Big(\alpha_n + \frac{1}{2}, \beta_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}(|w|-\theta))\Big)\,. \end{aligned} \tag{9}$$

Here c_1 and c_2 are integration constants and the mode-dependent parameters α_n and β_n are given by

$$\alpha_n = -\frac{1}{4} + \frac{1}{2}\sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \frac{1}{4}}\,,$$
$$\beta_n = -\frac{1}{4} - \frac{1}{2}\sqrt{\left(\frac{m_n}{\tilde{k}}\right)^2 + \frac{1}{4}}\,. \tag{10}$$

Similar expressions have been obtained in the cases of graviton [5] and fermions [17]. Below, we shall consider the case of symmetric configuration, i.e., $w_L = 2\theta$ for simplicity. In this case, we have only to consider even and odd functions with respect to the minimum of the warp factor. In the case of the odd functions, we have $c_1 = 0$ while the even functions, we have $c_2 = 0$. Indeed

$$\rho^{(n)}_{even}(w) = c_1 F\Big(\alpha_n, \beta_n, \frac{1}{2}; \sin^2(\tilde{k}(|w| - \theta))\Big)\,,$$
$$\rho^{(n)}_{odd}(w) = c_2 |\sin\tilde{k}(|w| - \theta)| F\Big(\alpha_n + \frac{1}{2}, \beta_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}(|w| - \theta))\Big)\,. \tag{11}$$

In order to obtain the mass spectrum, we have to impose the boundary conditions. In this case they should be given as follows;

$$\rho^{(n)\prime}_{even}(0) = \rho^{(n)\prime}_{even}(2\theta) = \rho^{(n)\prime}_{odd}(0) = \rho^{(n)\prime}_{odd}(2\theta) = 0\,, \tag{12}$$

where the prime denotes the derivative against z.

We evaluate eqs. (11) under the boundary conditions (12). For the odd modes, we find

$$-\frac{1}{3}\left(\frac{m}{k}\right)^2 \sinh^2(kz_0) F\Big(\alpha + \frac{3}{2}, \beta + \frac{3}{2}, \frac{5}{2}; \tanh^2(kz_0)\Big) + $$
$$+ F\Big(\alpha + \frac{1}{2}, \beta + \frac{1}{2}, \frac{3}{2}; \tanh^2(kz_0)\Big) = 0\,. \tag{13}$$

Here we neglected the suffix n and used the relation $\sin(\tilde{k}\theta) = \tanh(kz_0)$.

On the other hand, for the even modes

$$F\Big(\alpha + 1, \beta + 1, \frac{3}{2}; \tanh^2(kz_0)\Big) = 0\,. \tag{14}$$

Now let us evaluate the eqs. (13) and (14) at large kz_0 limit to obtain the mass spectrum. As the result, we can analytically find the following expressions for the mass of the KK mode. For odd modes

$$m_n = 2\sqrt{(n+1)\left(n + \frac{3}{2}\right)}\, k e^{-kz_0}\,, \tag{15}$$

where we have used an useful formula of the hypergeometric function

$$F(\alpha, \beta, \gamma; 1) = \frac{\Gamma(\gamma)\Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha)\Gamma(\gamma - \beta)} \tag{16}$$

$\Gamma(x)$ is the Euler's gamma function.

On the other hand, the KK mass spectrum for even modes at $kz_0 \gg 1$ is obtained as

$$m_n = 2\sqrt{(n+1)\left(n+\frac{1}{2}\right)}ke^{-kz_0} \tag{17}$$

Note here that this behavior of the mass spectrum is quite different from the case of graviton [5]. We find no ultralight Kaluza-Klein modes in the spectrum of the gauge field. The reason seems to be that in the present case, the bulk gauge field corresponds to $\nu = 1/2$ in ref. [17].

What happens in the asymmetric case where $w_L \neq 2\theta$? In this case, we also expect that the qualitative property still holds, i.e., the KK mass spectrum becomes the order of ke^{-kz_0}. But we have one more parameter in this case, which is the distance between the two AdS_4 branes, w_L.

To conclude, in this paper we studied the behavior of the bulk gauge fields in the bigravity model. We solved the equations of motions for the Kaluza-Klein gauge fields and obtained the mass spectrum at large distance (kz_0) limit. As the result, we had no ultralight Kaluza-Klein modes in the mass spectrum unlike the case of graviton.

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